SOLID STATE PHYSICS I

EXAM March 2002

◇ Do not forget to write your full name and student number on each sheet.
◇ Please use separate sheets for each of the problems.
◇ The answers may be given in dutch

Problem 1

Some materials consist of parallel one-dimensional (1D) chains, with a 1D electron density $n = N/L$, where N denotes the total number of electrons within a chain length L. Consider these chains as an ideal free electron gas with electron mass m_e, and spin S.

a) Give the dispersion relation $\epsilon(k)$ for the electrons.
b) Derive an expression for the Fermi-energy ϵ_F.
c) Derive an expression for the electronic density of states $D(\epsilon)$, expressed in terms of ϵ, n, and ϵ_F.
d) Derive an expression for the ground state energy U_0 (at $T = 0$) of the electron gas, expressed in terms of U_0, n, and ϵ_F.
e) The pressure of the electron gas is $p = -\partial U_0/\partial L$. How does the pressure depend on the density n ?
f) Discuss the consequence of the electronic pressure on the crystal structure.
g) Consider chains formed by atoms with interatomic distance a along the chains, where each atom donates one conduction electron.

   g1) Sketch the bandstructure for this case in the reduced zone scheme, and indicate the Fermi-level.

   g2) Describe how electron-lattice interactions could lead to a metal-insulator transition in these chains.

Problem 2

a) What is, using Hund's rules, the ground state of the following free ions:

   i) V^{4+}, configuration $3d^1$.
   ii) Fe^{3+}, configuration $3d^5$.
   iii) Ho^{3+}, configuration $4f^{10}$

   Use spectroscopic notation for your answer (Example Eu^{2+}: $^8S_{7/2}$).

b) The oxide UO_2 shows the full spin+orbital paramagnetism expected for the $^3H_4$ Hund's rule state of U^{4+} (configuration $5f^2$). In VO_2 one observes that the magnetic moment is *not* equal to the Hund's rule value $p = g\sqrt{J(J+1)}$, but equal to

$p = 2\sqrt{S(S+1)}$ with $S = 1/2$. Explain why there is no orbital contribution in vanadium-dioxide.

c) The susceptibility of a paramagnetic substance is given by the Curie law

$$\chi = \frac{M}{B} = \frac{C}{T}$$

Consider a ferromagnet assuming the effective medium approximation. The surrounding paramagnetic atoms cause an exchange field $B_E = \lambda M$ acting on every moment, in addition to the applied field B_a. Use Curie's law to derive an expression for the effective susceptibility in mean field approximation.

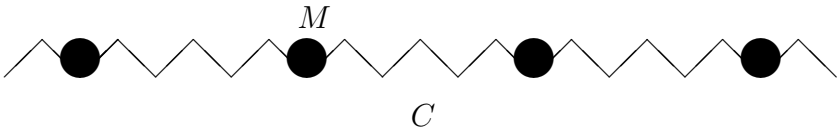
d) The mean field parameter for iron is $\lambda \approx 2045$, and the Curie constant is $C = 0.51$ K. At what temperature does iron order ferromagnetically.

e) Sketch the spontaneous magnetization of Iron as a function of temperature.

f) What are magnons ? Sketch the dispersion relation.

Problem 3

Consider a linear chain consisting of identical atoms with mass M, connected by identical springs with spring constant C (see figure). Assume that each atom interacts with its nearest-neighbour atom only, and that this interaction is linear in the relative displacement along the chain.



c) What is the difference between optical and acoustical modes ? Does this chain have optical modes ?

b) Give the equation of motion of the atoms as a function of their displacement along the chain.

c) Calculate the phonon dispersion relation, and make a sketch of this.

d) Give an expression for the sound velocity along the chain.

e) Describe the physical meaning of the Debye temperature. Give an expression for the Debye temperature in the linear chain.

f) Derive an expression for the total phonon energy at low temperatures within the Debye approximation, and show that the heat capacity at low temperatures is linear in the temperature.

$$note: \qquad \int_0^\infty \frac{x}{e^x - 1}\, dx = \frac{\pi^2}{6}$$